USN | | | | | | | | | |

07MCA12

First Semester MCA Degree Examination, December 2010
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Let U be the set of all real numbers,
$A = \{x : x \text{ is a solution of } x^2 - 1 = 0\}$
$B = \{-1, 4\}$. Compute,
i) $\overline{A \cap B}$ ii) $\overline{A \cup B}$ iii) $\overline{A}$ iv) $\overline{B}$. **(06 Marks)**

b. A survey of 500 TV watchers produced the following information : 285 watch football game, 195 watch hockey game, 115 watch basket ball game, 45 watch football and basket ball games, 70 watch football and hockey games, 50 watch hockey and basket ball games and 50 do not watch any of the 3 kinds of games. How many people in the survey watch,
i) all 3 kinds of games?
ii) exactly one of the games?
iii) at most 2 games?
iv) anyone of the games? **(08 Marks)**

c. By mathematical induction method, prove that 3 divides $(n^3 - n)$ for every integer $n \geq 2$. **(06 Marks)**

2 a. Define a compound proposition. Show that for any 3 proposition p, q and r, the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(08 Marks)**

b. Define conjunctive and disjunctive normal forms of a compound proposition, with one example. **(06 Marks)**

c. Write down the following propositions in symbolic forms and find their negations :
i) "All integers are rationales and some numbers are not integers".
ii) "If all triangles are right angled, then no triangle is equiangular". **(06 Marks)**

3 a. Define the following, with one example each :
i) Irreflexive relation
ii) Asymmetric relation
iii) Antisymmetric relation. **(06 Marks)**

b. Let $A = \{2, 3, 6, 12\}$ and let R and S be relations on A such that xRy iff $2|(x - y)$ and xSy iff $3|(x - y)$. Compute
i) $\overline{R}$ ii) S^{-1} iii) $R \cap S$ iv) $\overline{R \cup S}$. **(08 Marks)**

c. Define an equivalence relation on a set A. Let $S = \{1, 2, 3, 4, 5\}$ and $A = S \times S$. Define the relation R on A by $(a, b)\, R(a', b')$ iff $ab' = a'b$. Show that R is an equivalence relation. **(06 Marks)**

4 a. Let $S = \{a, b, c\}$ and $A = P(s)$, the power set of S on A. Define the relation R, by XRY if $X \subseteq Y$. Show that this relation is a partial order on A. Draw the Hasse diagram of R. **(08 Marks)**

b. Prove that a paset has at most one greatest element and at most one least element. **(04 Marks)**

c. Let $(L, \leq)$ be a lattice. Then, for any a, b and $c \in L$, show that
i) $a \vee (a \wedge b) = a$
ii) $a \vee (b \vee c) = (a \vee b) \vee c$
iii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$. **(08 Marks)**

5 a. Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$ and $C = \{c_1, c_2\}$ and $D = \{d_1, d_2, d_3, d_4\}$. Consider the following functions, from A to B, A to D, B to C, and D to B respectively

$f_1 = \{(a_1, b_2), (a_2, b_3), (a_3, b_1)\}$

$f_2 = \{(a_1, d_2), (a_2, d_1), (a_3, d_4)\}$

$f_3 = \{(b_1, c_2), (b_2, c_2), (b_3, c_1)\}$

$f_4 = \{(d_1, b_1), (d_2, b_2), (d_3, b_1)\}$.

Determine whether each function is one to one, whether each function is onto, and whether each function is everywhere defined. Justify. **(08 Marks)**

b. State pigeonhole principle, show that if 30 dictionaries in a library contain a total of 61, 327 pages, then one of the dictionaries must have at least 2045 pages. **(06 Marks)**

c. Define countable and uncountable sets, with examples. **(06 Marks)**

6 a. Define a simple graph and multiple graphs. Give an example for a graph which is not simple. **(06 Marks)**

b. If a graph G has more than two vertices of odd degree, then show that there can be no Euler path in G. **(06 Marks)**

c. Draw the following graphs

i) A complete graph with five vertices

ii) A three regular graph with eight vertices

iii) An Euler graph with twelve vertices

iv) A bipartite graph. **(08 Marks)**

7 a. Explain Konigsberg bridge problem. **(06 Marks)**

b. Show that there is one and only one path between every pair of vertices in a tree T. **(06 Marks)**

c. Show that every tree with two or more vertices is 2 – chromatic. Give an example to show that not every 2 – chromatic graph is a tree. **(08 Marks)**

8 a. Define an abelian group. Prove that a group G is abelian if and only if $(a\,b)^2 = a^2b^2$ for all $a, b \in G$. **(08 Marks)**

b. Define a normal subgroup of a group G. Prove that every subgroup of an abelian group is a normal subgroup. **(06 Marks)**

c. Define homomorphism. Let G be a group. Show that $f : G \rightarrow G$ defined by $f(x) = x^2$ is a homomorphism iff G is abelian. **(06 Marks)**

* * * * *